

General Disclaimer

One or more of the Following Statements may affect this Document

- This document has been reproduced from the best copy furnished by the organizational source. It is being released in the interest of making available as much information as possible.
- This document may contain data, which exceeds the sheet parameters. It was furnished in this condition by the organizational source and is the best copy available.
- This document may contain tone-on-tone or color graphs, charts and/or pictures, which have been reproduced in black and white.
- This document is paginated as submitted by the original source.
- Portions of this document are not fully legible due to the historical nature of some of the material. However, it is the best reproduction available from the original submission.

NASA Technical Memorandum 79029

(NASA-TM-79029) THE EFFECT OF NITROGEN ION
(⁺) IMPLANTATION ON THE FRICTION AND WEAR
CHARACTERISTICS OF IRON (NASA) 18 p HC
A02/MF A01

N79-12201

CSCL 11F

Unclas
37988

G3/26

THE EFFECT OF NITROGEN ION (N^+)
IMPLANTATION ON THE FRICTION AND
WEAR CHARACTERISTICS OF IRON

William R. Jones, Jr. and John Ferrante
Lewis Research Center
Cleveland, Ohio

November 1978



SUMMARY

The effect of implantation of nitrogen ions (N^+) on the friction and wear characteristics of pure iron sliding against M-50 steel (unimplanted) was studied using a pin-on-disk sliding friction apparatus. Test conditions included: a dry air atmosphere, 1/2 kilogram load (4.9 N), sliding velocities of 0.043 to 0.051 meters per second (~ 15 -18 rpm), a U. S. P. mineral oil lubricant, and a nitrogen ion implantation dose of 5×10^{15} ions per centimeter squared.

The friction and wear properties of pure iron sliding against M-50 steel were not significantly altered after nitrogen implantation. The unimplanted iron exhibited an average wear rate of $1.47 \pm 0.27 \text{ M}^3/\text{N-M}$ ($\times 10^{-14}$) compared to 1.53 ± 0.73 for the nitrogen implanted iron. Average friction coefficients were 0.10 (unimplanted) and 0.09 (implanted).

INTRODUCTION

Ion implantation (ref. 1) is the process by which elements are injected into the surface region of a solid. This is accomplished by accelerating ions of the injected element in a vacuum chamber ($\sim 10^{-5}$ torr) and allowing them to strike the solid substrate. Ion energies are usually in the range of 10 to 500 keV.

The most important application of ion implantation has been in the semiconductor industry to introduce dopants into the semiconductor (ref. 2). More recently, studies have shown that the implantation of certain elements (Cr, He, and B) can improve the corrosion resistance of steels and other alloys (refs. 3-5). Other applications have been in the areas of catalysis (ref. 6) and fatigue (ref. 7).

Finally, a number of investigators have reported substantial reductions in friction and wear of implanted surfaces. Mo and S implanted into the same steel surface reduced friction by 30% (ref. 8). Implantation of B^+ , N^+ , and Mo^+ reduced wear of nitriding steel by more than a factor of ten (ref. 9). Similar results were obtained with nitrogen and carbon implanted in several different steels (ref. 10). Implantation has even been reported to improve hydrodynamic lubrication (ref. 7).

However, in most of the above studies, there has been little attempt to control experimental conditions and materials. Therefore, the present investigation had two main objectives. The first was to establish a controlled testing procedure to determine the effects of ion implantation. The second objective was to use this procedure to study the effects of nitrogen ion implantation on the friction and wear properties of pure iron sliding against M-50 steel in dry air under lubricated conditions in a pin-on-disk apparatus. Conditions included: a 1/2 kilogram load (4.9 N), 0.043 to 0.051 meters per second sliding velocities, a U. S. P. mineral oil lubricant and a nitrogen ion dose of 5×10^{15} ions per square centimeter. This low dose will help set lower bounds on the threshold of ion implantation effects on wear.

APPARATUS

The pin-on-disk sliding friction apparatus is shown in figure 1. The test specimens were contained inside a plastic chamber. This allowed the moisture content of the test atmosphere to be controlled. A stationary 0.476-centimeter radius hemispherically tipped iron rider was placed in sliding contact with a rotating 6.3-centimeter diameter (1.2 cm thick) steel disk. A constant sliding speed in the range of 0.043 to 0.051 meters per second was maintained. A normal load of 1/2 kilogram (4.9 N) was applied with a deadweight.

MATERIALS

The riders were machined from polycrystalline iron rod (99.95%) and fully annealed prior to testing. The disks were made of CVM M-50 steel having a Rockwell C hardness of 62 to 64. The lubricant was a commercial U. S. P. oil with the properties listed in table I.

TEST PROCEDURE

Disk specimens were ground and lapped to a surface finish of 10×10^{-8} meter (4μ in.) Ra. They were then scrubbed with a paste of levigated alumina and water. Riders were cleaned similarly except a commercial (nonabrasive) detergent was used instead of alumina. All specimens were dried on clean filter paper.

The specimens were assembled in the test chamber. Approximately 50 ml of lubricant was added to the lubricant cup. The chamber was purged with dry air (<50 ppm H_2O) for a minimum of 10 minutes. The disk was set in motion and the rider loaded against it. Frictional force was measured by a strain gage. Rider wear was determined periodically by stopping the test and measuring the wear scar diameter. In addition to wear tests, diamond pyramid microhardness measurements were made at a 150 gm load. Hardness measurements were made on implanted and unimplanted iron before tests and in the wear scar on both materials after wear tests.

RESULTS AND DISCUSSION

An attempt was made in this investigation to perform ion implantation studies under carefully controlled conditions. The pin-on-disk apparatus was selected since it is a classic technique for examining wear and is relatively easy to reproduce in any laboratory. The apparatus was placed in a plastic chamber which enabled control over the environment. The sliding speed and load were selected to be sufficiently low such that elastohydrodynamic effects were minimized and the experiments were in the boundary lubrication regime. Mineral oil was chosen as a lubricant since it is readily available and well characterized. Finally the rider (pin) material was standardized as pure iron to eliminate effects from varying materials. The riders (pins) were polished, annealed and cleaned under fixed conditions in order to eliminate the effects of material preparation on the results. The disk material was selected to have a high hardness in order to make rider wear the controlling factor.

FRICTION AND WEAR

Table II shows the friction and wear data and conditions for each individual test. A total of six baseline or unimplanted tests and eight implanted tests were performed. All tests were run with a 1/2 kilogram load (4.9 N). Tests were performed at approximately 15-18 rpm which, depending on the disk wear track circumference, yielded sliding velocities in the range 0.043 to 0.051 meters per second.

Table III summarizes the friction and wear results for the two situations (implanted and unimplanted). As can be seen the wear rate for the implanted iron is almost the same as the control (unimplanted) tests.

Rider wear volume is shown as a function of sliding distance in figure 2 (unimplanted) and figure 3 (implanted). There is considerably more data scatter for the implanted situation. This of course, results in a much larger standard deviation.

The data of figures 2 and 3 has been replotted in a different format in figures 4 and 5. Here incremental wear rates between each wear measurement are plotted as a function of sliding distance. In this format, any early implantation effects on wear should be evident. Again, except for more scatter in the implanted results (fig. 5) no obvious differences are seen.

Two statistical analyses were done on the wear data presented. The wear rates were essentially the same for the implanted and the unimplanted pins. The scatter in the implanted wear rates was much higher than in the unimplanted. Statistical analysis of the wear rates using a student distribution (ref. 14) indicated that there was no significant statistical differences in wear rates for the two cases. Statistical analysis of the standard deviations using a Chi-squared distribution (ref. 14), however, indicated that there was a 90% confidence level that the implanted standard deviation was outside the unimplanted. This is a difficult result to interpret since it would indicate that implantation at these levels could cause changes in either direction but in the mean would have no significant effect.

CORRELATION WITH OTHER INVESTIGATORS

Although the literature is replete with effects of ion implantation on friction and wear, basically only three groups have performed tribological experiments. Pavlov et. al. (ref. 11) reported large increases (2 to 6 times) in the coefficient of friction for 40 keV Ar^+ ion bombarded steel for doses to 9×10^{17} ions/cm². Substantial improvements in wear for implanted aluminum rubbing against steel cylinders was also reported but the implantation and wear conditions were not specified.

Hartley et. al. (ref. 9) at Harwell have published the effects of a variety of implanted ions on the coefficient of friction of En352 steel. Both increases and decreases were noted. Large reduction (up to 10 times) in wear occurred for a 440C steel pin sliding against a mild steel disk (N^+ implanted to 10^{18} ions/cm²). More recently the relative decrease in wear rate for nitrogen (50 keV) implanted nitriding steel as a function of dose was reported (ref. 12). Little change was noted at a dose of 4×10^{16} ions/cm². But above this dose, wear decreased with increasing dose until a maximum decrease (~30 times) occurred at a dose of 3×10^{17} ions/cm². Thus, based on these results, it is not surprising that implantation effects were not observed in the present study since the total dose of 5×10^{15} ions/cm² was less than one-tenth of the threshold reported above.

The third group to publish in this area is at the Naval Research Laboratory. They have reported (ref. 13) implantation effects on the sliding wear of 416 stainless steel and AISI 52100 steel. Either a ball-on-cylinder or crossed-cylinder-on-cylinder geometry was used. Forty keV nitrogen ions were implanted to a dose of 10^{17} ions/cm². For the 52100 steel tests a factor of two improvement in wear was found. Much greater decreases (25 to 50 times) were observed for the 416 stainless steel tests.

These effects were attributed to inward migration of the nitrogen ions during the wear process. The results were not believed to be caused by the formation of surface nitrides since the depth of wear was much greater than the penetration depth of the implanted nitrogen. Radiation damage was also ruled out because argon implantation to the same doses did not yield any wear reductions.

Again, these results indicate that the dose of the present study was probably too low to produce any gross changes in the tribological properties which establishes a lower bound for iron. It should be noted that the low dose obtained for the present work was much lower than had initially been sought. Because of relatively low beam currents and the fact that the beam was rastered, doses greater than 10^{16} ions/cm² would have required a prohibitively long exposure time.

MICROHARDNESS MEASUREMENTS

We obtained microhardnesses on implanted and unimplanted iron of 81 and 58 kg/mm², respectively, with an average scatter of $\pm 10\%$. In the wear scars we obtained 111 kg/mm² in the implanted and 102 kg/mm² unimplanted. Thus, although there were differences initially, strain hardening in the wear tests produced approximately the same final hardnesses.

AES ANALYSIS OF IMPLANTED SURFACES

The results of AES (Auger Electron Spectroscopy) analysis of the implanted iron bullets are presented in figure 6. In figure 6 we have a spectrum showing the surface elemental analysis. In figure 7 we present the portion of the spectrum presented in 6 limited to the location of the nitrogen peak. The composition as a function of depth was determined by depth profiling combining sputtering with AES. The sputtering rate was calibrated by determining the time needed to remove a 1000 Å Tantalum Oxide film from Tantalum.

The purpose of depth profiling the Auger analysis was to detect and to attempt to determine the spatial distribution of the implanted nitrogen. As can be seen with no sputtering the surface was primarily composed of sulfur, carbon, oxygen and iron. The carbon contamination is common and is a result of handling the specimen in air. As can be seen on this sensitivity scale there is no nitrogen present in the surface films. Sputtering rapidly removes the carbon and after removal of 500 Å we have basically an iron oxide surface, whereas at 7500 Å removed we have basically

a pure iron surface with a small amount of carbon monoxide adsorbed from the ambient in the vacuum system whose pressure was typically in the low 10^{-9} torr range. In figure 7 we show the comparable nitrogen concentration as a function of depth. In order to detect the nitrogen it was necessary to greatly increase the amplification. We can see that on the surface and in the surface oxide there is a greater quantity of nitrogen (peak ht. $\sim$ quantity) than within the bulk of the material. An examination of an unimplanted bullet revealed that the distribution of nitrogen observed in the implanted bullet was the same. Thus we must conclude that the nitrogen detected is that which would naturally occur from the processing of the bullet.

This seemingly negative result has relevance regarding the distribution of nitrogen in the implanted bullet. The sensitivity of AES is approximately 1 part in 10^3 for nitrogen. For the doses administered and assuming the nitrogen to be uniform distributed over 1 micron in depth we would have 1 part in 10^4 which is below the level of detectability. The estimate for the range of implantation is 0.3 ± 0.1 micron. Thus we can conclude from these results that there is no concentration of nitrogen to a region smaller than 0.1 micron or less since this spatial distribution of implantation would be detectable by Auger spectroscopy.

SUMMARY OF RESULTS

The effect of nitrogen ion (N^+) implantation on the friction and wear characteristics of pure iron sliding against M-50 steel (unimplanted) was studied using a pin-on-disk sliding friction apparatus under carefully controlled conditions. Test conditions included a dry air atmosphere, 1/2 kilogram load (4.9 N), sliding velocities of 0.043 to 0.051 meters per second (~ 15 -18 rpm) a U.S. P. mineral oil lubricant, and a nitrogen ion implantation dose of 5×10^{15} ions per square centimeter. The major results were as follows:

1. The friction and wear characteristics of pure iron (implanted) sliding against M-50 steel (unimplanted) were not significantly altered as a result of the nitrogen implantation.

2. The concentration of nitrogen in the near surface region (0-15000 Å) of the implanted iron (as detected by Auger spectroscopy) was similar to the amount found in the unimplanted iron.

REFERENCES

1. Townsend, Peter D.; Kelly, J. C.; and Hartley, N. E. W.: Ion Implantation, Sputtering, and Their Applications. Academic Press (London), 1976.
2. Mayer, James W.; Erikson, Lennart; and Davies, John A., Ion Implantation in Semiconductors, Academic Press, 1970.
3. Ashworth, V.; et. al.: Effect of Ion Implantation on the Corrosion Behavior of Iron. Ion Implantation in Semiconductors and Other Materials, S. Namba, ed., Plenum Press, 1975, pp. 367-373.
4. Khirnyi, Yu M.; and Solodovnikov, A. P.: Effect of Increased Corrosion Resistance in Metals Irradiated with Helium Ions. Sov. Phys. Dokl., vol. 19, 1974, p. 31.
5. Rickards, J.; and Dearnaley, G.: Ion Implantation and Backscattering From Oxidized Single-Crystal Copper. Applications of Ion Beams to Metals, S. T. Picraux, E. P. Eernisse, and F. L. Vook, eds., Plenum Press, 1974, pp. 101-109.
6. Grenness, M.; Thompson, M. W.; and Cahn, R. W.: J. Appl. Electrochem. vol. 4, no. 3, 1974, pp. 211-218.
7. Hartley, N. E. W.: Tribological Effects in Ion-Implanted Metals. Inst. Phys. Conf. Ser. No. 28, 1976, Ch. 5., pp. 210-223.
8. Hartley, N. E. W.; Dearnaley, G.; and Turner, J. F.: Frictional Changes Induced by the Ion Implantation of Steel. Ion Implantation in Semiconductors and Other Materials, B. L. Crowder, ed., Plenum Press, 1973, pp. 423-436.

9. Picraux, S. T.; Eernisse, E. P.; and Vook, F. L., eds.: Applications of Ion Beams to Metals. Plenum Press, 1974, p. 123.
10. Dearnaley, G.; and Hartley, N. E. W.: Proc. Conference "Ion Plating and Allied Techniques," Edinburgh, CEP Consultants, 1977.
11. Pavlov, A. V.; et al.: Proc. All-Soviet Meeting on Ion Beam Physics, Kiev, 1974.
12. Hartley, N. E. W.: Ion Implantation Case Studies - Manufacturing Applications. Presented at the Institution & Metallurgists Spring Review Course, Surface Treatments for Protection, University College (Cardiff), Apr. 7-10, 1978.
13. Hirvonen, J. K.: Ion Implantation in Tribology and Corrosion Science. J. Vac. Sci. Technol., vol. 15, no. 5, Sep.-Oct., 1978, pp. 1662-1668.
14. Spiegel, Murray R.: Schaum's Outline of Theory and Problems of Statistics, Itatistics. Schaum Publishing Co., 1961.

TABLE I. - TYPICAL LUBRICANT PROPERTIES

Lubricant type	U. S. P. mineral oil
Viscosity, N-sec/m ² (Cp)	
37.8° C	0.06 (60)
98.9° C	0.007 (7)
Specific gravity	
15.6° C	0.888
25° C	0.883

TABLE II. - FRICTION AND WEAR RESULTS

Test	Implanation species	Speed m/s	Average coefficient of friction	Average wear rate after run-in
1	None ↓	0.044	0.10	1.26×10^{-14}
2		.043	.11	1.17×10^{-14}
3		.046	.12	1.49×10^{-14}
4		.045	.09	1.47×10^{-14}
5		.051	.10	1.95×10^{-14}
6		.051	.09	1.49×10^{-14}
7	N ₂ ↓	.044	----	1.57×10^{-14}
8		.045	----	1.34×10^{-14}
9		.051	0.09	2.69×10^{-14}
10		.048	.09	2.56×10^{-14}
11		.049	.08	$.94 \times 10^{-14}$
12		.045	.09	$.63 \times 10^{-14}$
13		.047	.09	1.16×10^{-14}
14		.045	.09	1.31×10^{-14}

TABLE III. - SUMMARY OF FRICTION AND WEAR RESULTS

Implanation species	Number of tests	Average coefficient of friction	Average wear wear rate $M^3/N-M \times 10^{-14}$	Standard deviation
None	6	0.10	1.47	± 0.27
Nitrogen	8	.09	1.53	± 0.73

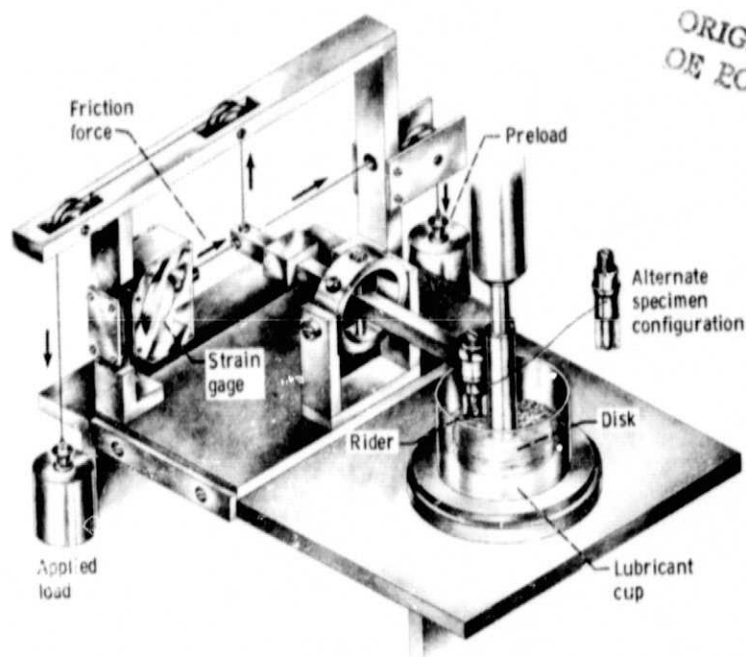


Figure 1. - Pin-on-disk sliding friction apparatus.

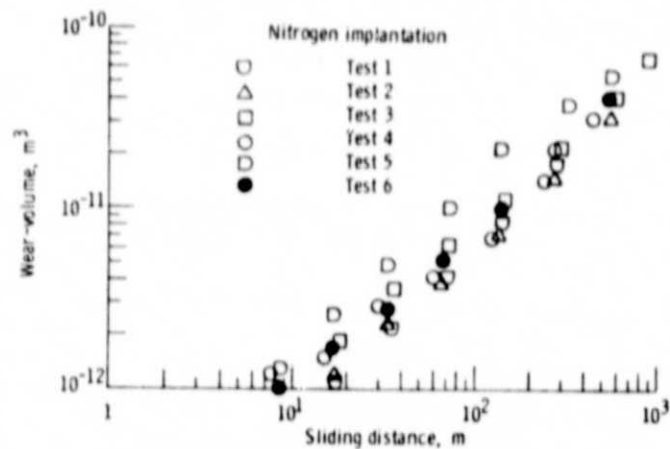


Figure 2. - Rider (unimplanted) wear volume as a function of sliding distance.

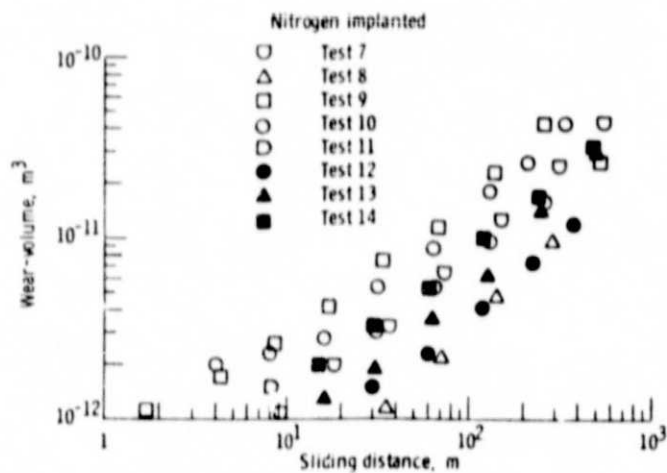


Figure 3. - Rider (nitrogen implanted) wear volume as a function of sliding distance.

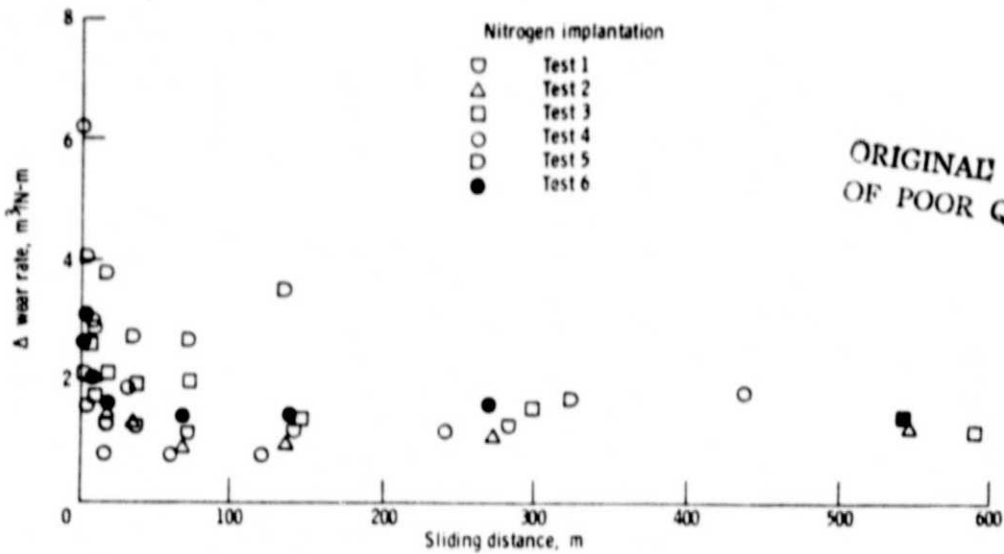


Figure 4. - Incremental wear rate as a function of sliding distance.

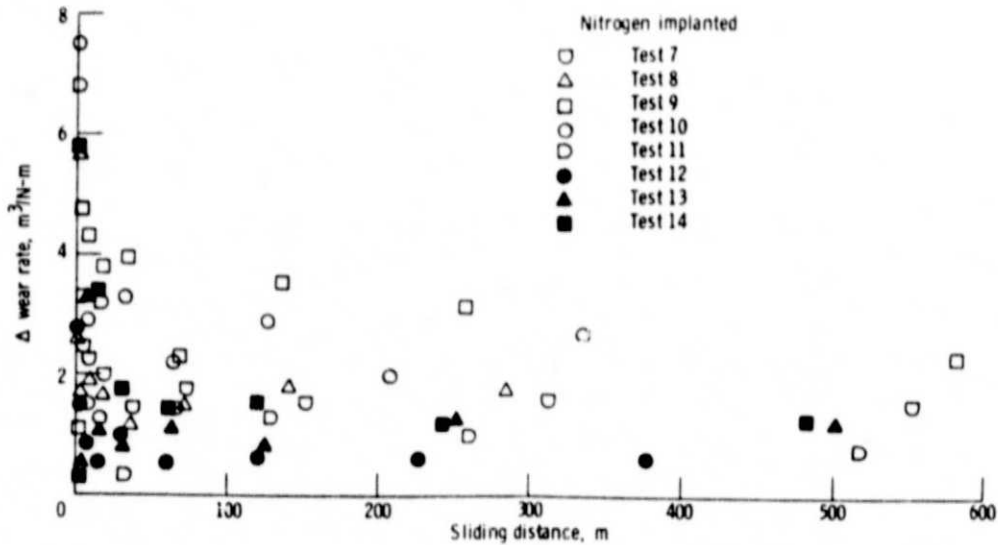


Figure 5. - Incremental wear rate as a function of sliding distance.

ORIGINAL PAGE IS
OF POOR QUALITY

ORIGINAL PAGE IS
OF POOR QUALITY

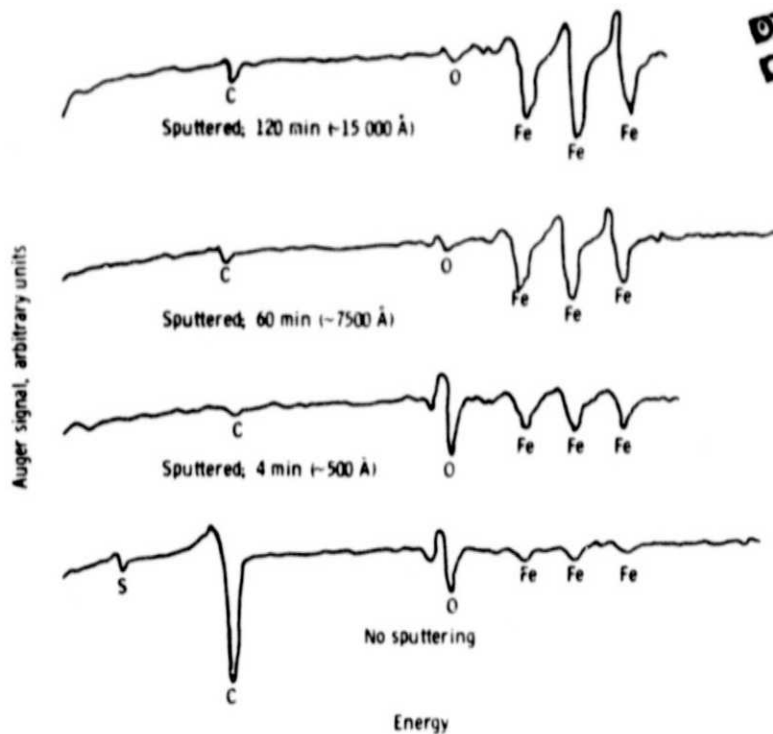


Figure 6. - Auger spectrum of nitrogen implanted iron surface after various sputtering intervals. (Beam energy, 2000 eV; beam current, 1 μ A.)

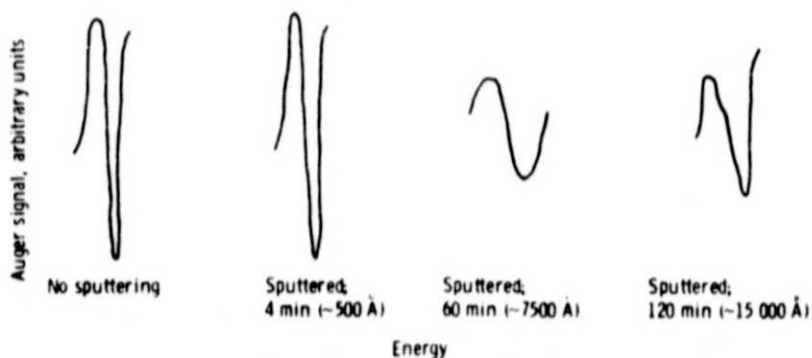


Figure 7. - Nitrogen Auger peak from nitrogen implanted surface after various sputtering intervals. (Beam energy, 2000 eV; beam current, 1 μ A.)

Table 4. Comparison of the stress intensity factors for isotropic and orthotropic strips with a symmetrically located internal crack. Tension: $\sigma_{yy} = \sigma_{xx}(\bar{x}, y)$, bending: $\sigma_{xx} = \sigma_b(1-2y/h)$, $(b-a)/(h/\cos\theta) = 0.6$, $c = (b-a)/2$, $a = (h/\cos\theta) - b$

	$\theta = 0$	$\theta = \pi/6$			
	Tension	Tension		Bending	
	$k_1/\sigma_m\sqrt{c}$	$k_1/\sigma_m\sqrt{c}$	$k_2/\sigma_m\sqrt{c}$	$k_1/\sigma_b\sqrt{c}$	$k_2/\sigma_b\sqrt{c}$
Isotropic	1.303	1.080	0.504	0.248	0.137
Ortho. (30°)	1.226	1.420	0.553	0.288	0.141
Ortho. (120°)	1.226	1.172	0.518	0.258	0.138

by 120° E_{11} axis makes 120° with the x-axis, i.e., in the latter case the material has been rotated by 90° (see Figure 1). The isotropic results are also given in the table. The table shows that in the inclined crack problem not only the material orthotropy but also the orientation of the axes of orthotropy may have a significant effect on the stress intensity factors.

In the case of an edge crack, i.e., for $a=0$, $b < h/\cos\theta$, the integral equations (34) remain unchanged. However, the unknown functions $f_1(t)$ and $f_2(t)$ are bounded at $t=0$ and the conditions (36) are no longer valid. In this case the integral equations can be solved numerically by first normalizing the interval $(0,b)$ to $(-1,1)$ through the change in variables

$$t = \frac{b}{2} (r+1) \quad , \quad x_2 = \frac{b}{2} (s+1) \quad , \quad -1 < (s,r) < 1 \quad , \quad (50)$$

and then using again a Gauss-Chebyshev integration formula. A convenient technique in this problem is defining the unknown functions by

$$f_i(t) = G_i(r)/\sqrt{1-r^2} \quad , \quad i=1,2 \quad (51)$$

and using the collocation points s_j obtained from $U_{n-1}(s_j) = 0$, ($j=1, \dots, n-1$) and the condition $G_i(-1) = 0$ (to account for boundedness of $f_i(t)$ at $t = 0$) to calculate $G_i(r_k)$, ($k=1, \dots, n$) $T_n(r_k) = 0$, where T_n and U_n are Chebyshev polynomials. Table 5 shows the calculated results for the edge crack. In this problem too the external load is either a uniform tension or a uniform bending applied to the strip away from the crack region.

The results show that depending on the crack geometry, under bending and under uniform crack surface shear traction at one of the crack tips the mode I component of the stress intensity factor k_I may be negative. In this case the crack surfaces would be partially closed and the problem would become a crack-contact problem with the length of the contact region being unknown. Therefore, taken separately, the solutions given in this paper for which $k_I < 0$ are not valid. However, these solutions can be used under the combined loading conditions in which, in addition to the bending and the transverse shear, there is a sufficiently large membrane component of the external load such that the superimposed mode I stress intensity factors at both crack tips are positive.

ACKNOWLEDGEMENTS

This work was partially supported by the National Science Foundation under the Grant ENG77-19127, by NASA under the Grant NGR39-007-011 and by the Technical University of Istanbul. The numerical calculations were carried out at the Computing Center of the Technical University of Istanbul.

Table 5. Stress intensity factors for an edge crack ($a=0$) in an orthotropic strip under tension or bending away from the crack region

$\frac{b}{h/\cos\theta}$	Tension: $\sigma_{xx}^\infty = \sigma_m$, $k_1' = k_1(b)/\sigma_m\sqrt{b}$, $k_2' = k_2(b)\sigma_m\sqrt{b}$						
	$\theta = 0$		$\theta = \pi/6$		$\theta = \pi/4$		$\theta = \pi/3$
	k_1'	k_2'	k_1'	k_2'	k_1'	k_2'	k_1' k_2'
0.1	1.13	0	1.19	0.32	0.99	0.38	0.66 0.35
0.2	1.32	0	1.38	0.34	1.16	0.41	0.79 0.57
0.3	1.61	0	1.70	0.39	1.42	0.47	0.98 0.44
0.4	2.04	0	2.34	0.47			
0.5	2.72	0	3.57	0.63			
0.6	3.86	0	6.09	0.97			
	Bending: $\sigma_{xx}(\bar{x}, y) = \sigma_b(1-2y/h)$, $k_1 = k_1(b)/\sigma_b\sqrt{b}$, $k_2 = k_2(b)/\sigma_b\sqrt{b}$						
	k_1'	k_2'	k_1'	k_2'	k_1'	k_2'	k_1' k_2'
	0.1	0.99	0	1.06	0.28	0.89	0.33 0.59
	0.2	1.01	0	1.09	0.24	0.92	0.29 0.64
	0.3	1.08	0	1.20	0.23	1.01	0.28 0.71
	0.4	1.21	0	1.51	0.23		
	0.5	1.43	0	2.13	0.27		
	0.6	1.81	0	3.33	0.41		

REFERENCES

1. Ang, D. D. and Williams, M. L., "Combined stresses in an orthotropic plate having a finite crack," Journal of Applied Mechanics, Vol. 28, Trans. ASME, 1961, pp. 372-378.
2. Savin, G. N., Stress Concentration Around Holes, Pergamon Press, New York, 1961.
3. Sih, G. C., Paris, P. C. and Irwin, G. R., "On cracks in rectilinearly anisotropic bodies," Int. J. Fracture Mechanics, Vol. 1, 1965, pp. 189-203.
4. Krenk, S., "Stress distribution in an infinite anisotropic plate with collinear cracks," Int. J. Solids and Structures, Vol. 11, 1975, pp. 449-460.
5. Delale, F. and Erdogan, F., "The problem of an internal and edge cracks in an orthotropic strip," Journal of Applied Mechanics, Vol. 44, Trans. ASME, 1977, pp. 237-242.
6. Krenk, S., "On the elastic strip with an internal crack," Int J. Solids and Structures, Vol. 11, 1975, pp. 693-708.
7. Lekhnitskii, S. G., Theory of Elasticity of an Anisotropic Elastic Body, Holden-Day, Inc. 1963.
8. Muskhelishvili, N. I., Singular Integral Equations, P. Noordhoff, Groningen, The Netherlands, 1953.
9. Erdogan, F. and Gupta, G. D., "On the numerical solution of singular integral equations," Quarterly of Applied Mathematics, Vol. 30, 1972, pp. 525-534.
10. Delale, F. and Erdogan, F., "Transverse shear effect in a circumferentially cracked cylindrical shell," NASA CR-145265, 1977

APPENDIX A

Expressions of the functions $R_j(s)$ and the solution of equations (33):

$$\begin{aligned}
 R_1(s) &= \Delta_1 \left\{ \left(\omega_1 n_1^2 - \frac{n_2^2}{\omega_1} \right) I_1^1 + \left(\frac{n_2^2}{\omega_2} - n_1^2 \omega_1 \right) I_2^1 + (n_1^2 \omega_1^3 - n_2^2 \omega_1) J_1^1 \right. \\
 &\quad \left. + (n_2^2 \omega_2 - n_1^2 \omega_2^3) J_2^1 + 2n_1 n_2 [\omega_1 K_1^1 - \omega_2 K_2^1 - \omega_1 L_1^1 + \omega_2 L_2^1] \right\} , \\
 R_2(s) &= \frac{\Delta_1}{i} \left\{ n_1 n_2 \left[\left(1 - \frac{1}{\omega_1} - \omega_1 \right) I_1^1 + \left(\frac{1}{\omega_2} + \omega_2 \right) I_2^1 - (\omega_1 + \omega_1^3) J_1^1 \right. \right. \\
 &\quad \left. \left. + (\omega_2 + \omega_2^3) J_2^1 \right] + (n_1^2 - n_2^2) [\omega_1 K_1^1 - \omega_2 K_2^1 - \omega_1 L_1^1 + \omega_2 L_2^1] \right\} , \\
 R_3(s) &= \Delta_1 \left\{ \left(n_1^2 \omega_1 - \frac{n_2^2}{\omega_1} \right) I_1^2 + \left(\frac{n_2^2}{\omega_2} - n_1^2 \omega_1 \right) I_2^2 + (n_1^2 \omega_1^3 - n_2^2 \omega_1) J_1^2 \right. \\
 &\quad \left. + (n_2^2 \omega_2 - n_1^2 \omega_2^3) J_2^2 + 2n_1 n_2 [\omega_1 K_1^2 - \omega_2 K_2^2 - \omega_1 L_1^2 + \omega_2 L_2^2] \right\} , \\
 R_4(s) &= \frac{\Delta_1}{i} \left\{ n_1 n_2 \left[-\left(\frac{1}{\omega_1} + \omega_1 \right) I_1^2 + \left(\frac{1}{\omega_2} + \omega_2 \right) I_2^2 - (\omega_1 + \omega_1^3) J_1^2 \right. \right. \\
 &\quad \left. \left. + (\omega_2 + \omega_2^3) J_2^2 \right] + (n_1^2 - n_2^2) [\omega_1 K_1^2 - \omega_2 K_2^2 - \omega_1 L_1^2 + \omega_2 L_2^2] \right\} ; \quad (A1-A4)
 \end{aligned}$$

$$\Delta_1 = \frac{1}{2\pi a_{22}(\omega_1^2 - \omega_2^2)s^2} ; \quad (A5)$$

$$I_j^k(s) = \int_a^b E_j^k(s, t) f_1(t) dt ,$$

$$J_j^k(s) = \int_a^b F_j^k(s, t) f_2(t) dt ,$$

$$K_j^k(s) = \int_a^b F_j^k(s, t) f_1(t) dt ,$$

$$L_j^k(s) = \int_a^b E_j^k(s, t) f_2(t) dt , \quad (j, k) = (1, 2) ; \quad (A6-A9)$$

$$\begin{aligned}
E_j^1(s, t) &= \pi e^{-|s| t n_1 \lambda_j} [n_1 \lambda_j \cos c_j t + c_j \sin |s| c_j t \\
&\quad + i \frac{s}{|s|} c_j \cos c_j s t - i n_1 \lambda_j \sin c_j s t] , \\
E_j^2(s, t) &= \pi e^{-|s| \lambda_j (h - n_1 t)} \{ -n_1 \lambda_j \cos [c_j s (t - n_1 h + n_1 \omega_j^2 h)] \\
&\quad + c_j \sin [|s| c_j (t - n_1 h + n_1 \omega_j^2 h)] \\
&\quad + i c_j \frac{s}{|s|} \cos [s c_j (t - n_1 h + \omega_j^2 n_1 h)] \\
&\quad + i n_1 \lambda_j \sin [s c_j (t - n_1 h + \omega_j^2 n_1 h)] \} , \quad j=1,2 ; \quad (A10, A11)
\end{aligned}$$

$$\begin{aligned}
F_j^1(s, t) &= e^{-|s| \lambda_j n_1 t} \left[-\frac{c_j}{\omega_j} \cos c_j s t + n_1 b_j \sin c_j |s| t \right. \\
&\quad \left. + i \frac{s}{|s|} n_1 b_j \cos c_j s t + i \frac{c_j}{\omega_j} \sin c_j s t \right] , \\
F_j^2(s, t) &= \pi e^{-|s| \lambda_j (h - n_1 t)} \left\{ \frac{c_j}{\omega_j} \cos [s c_j (t - n_1 h + \omega_j^2 n_1 h)] \right. \\
&\quad \left. + n_1 b_j \sin [|s| c_j (t - n_1 h + \omega_j^2 n_1 h)] \right. \\
&\quad \left. + i \frac{s}{|s|} n_1 b_j \cos [s c_j (t - n_1 h + \omega_j^2 n_1 h)] \right. \\
&\quad \left. - i \frac{c_j}{\omega_j} \sin [s c_j (t - n_1 h + \omega_j^2 n_1 h)] \right\} , \quad j=1,2 ; \quad (A12, A13)
\end{aligned}$$

$$\begin{aligned}
\lambda_j &= \omega_j / (n_1^2 \omega_j^2 + n_2^2) , \quad b_j = 1 / (n_1^2 \omega_j^2 + n_2^2) , \quad c_j = -n_2 / (n_1^2 \omega_j^2 + n_2^2) , \\
&\quad j=1,2 ; \quad (A14-A16)
\end{aligned}$$

Solution of equations (33):

$$C_k(s) = \frac{1}{\Delta(s)} \sum_{j=1}^4 m_{kj}(s) R_j(s) \quad , \quad k=1, \dots, 4 \quad ; \quad (A17)$$

$$\begin{aligned} \Delta(s) = & (r_1 - r_3)(r_1 - r_4)(e^{r_1 sh} - e^{r_2 sh})(e^{r_4 sh} - e^{r_3 sh}) \\ & - (r_1 - r_3)(r_1 - r_2)(e^{r_2 sh} - e^{r_3 sh})(e^{r_1 sh} - e^{r_4 sh}) \\ & + (r_1 - r_2)(r_1 - r_4)(e^{r_2 sh} - e^{r_4 sh})(e^{r_1 sh} - e^{r_3 sh}) \quad ; \quad (A18) \end{aligned}$$

$$\begin{aligned} m_{11}(s) = & r_4(r_3 - r_2)e^{(r_2 + r_3)sh} + r_3(r_2 - r_4)e^{(r_2 + r_4)sh} \\ & + r_2(r_4 - r_3)e^{(r_3 + r_4)sh} \quad , \end{aligned}$$

$$m_{12}(s) = (r_2 - r_3)e^{(r_2 + r_3)sh} - (r_2 - r_4)e^{(r_2 + r_4)sh} - (r_4 - r_3)e^{(r_3 + r_4)sh} \quad ,$$

$$m_{13}(s) = r_2(r_4 - r_3)e^{r_2 sh} + r_3(r_2 - r_4)e^{r_3 sh} + r_4(r_3 - r_2)e^{r_4 sh} \quad ,$$

$$m_{14}(s) = (r_3 - r_4)e^{r_2 sh} + (r_4 - r_2)e^{r_3 sh} + (r_2 - r_3)e^{r_4 sh} \quad ,$$

$$\begin{aligned} m_{21}(s) = & r_4(r_1 - r_3)e^{(r_1 + r_3)sh} - r_3(r_1 - r_4)e^{(r_1 + r_4)sh} \\ & + r_1(r_3 - r_4)e^{(r_3 + r_4)sh} \quad , \end{aligned}$$

$$\begin{aligned} m_{22}(s) = & (r_1 - r_3)e^{(r_1 + r_3)sh} + (r_1 - r_4)e^{(r_1 + r_4)sh} \\ & + (r_4 - r_3)e^{(r_3 + r_4)sh} \quad , \end{aligned}$$

$$m_{23}(s) = r_1(r_3 - r_4)e^{r_1 sh} - r_3(r_1 - r_4)e^{r_3 sh} + r_4(r_1 - r_3)e^{r_4 sh} \quad ,$$

$$m_{24}(s) = (r_4 - r_3)e^{r_1 sh} + (r_1 - r_4)e^{r_3 sh} - (r_1 - r_3)e^{r_4 sh} ,$$

$$m_{31}(s) = r_2(r_1 - r_4)e^{(r_1 + r_4)sh} + r_1(r_4 - r_2)e^{(r_2 + r_4)sh} \\ - r_4(r_1 - r_2)e^{(r_1 + r_2)sh} ,$$

$$m_{32}(s) = (r_1 - r_4)e^{(r_1 + r_4)sh} + (r_1 - r_2)e^{(r_1 + r_2)sh} \\ + (r_4 - r_1)e^{(r_1 + r_4)sh} ,$$

$$m_{33}(s) = r_1(r_4 - r_2)e^{r_1 sh} - r_2(r_1 - r_4)e^{r_2 sh} - r_4(r_1 - r_2)e^{r_4 sh} ,$$

$$m_{34}(s) = (r_2 - r_4)e^{r_1 sh} - (r_1 - r_4)e^{r_2 sh} + (r_1 - r_2)e^{r_4 sh} ,$$

$$m_{41}(s) = r_3(r_1 - r_2)e^{(r_1 + r_2)sh} - r_2(r_1 - r_3)e^{(r_1 + r_3)sh} \\ + r_1(r_2 - r_3)e^{(r_2 + r_3)sh} ,$$

$$m_{42}(s) = - (r_1 - r_2)e^{(r_1 + r_2)sh} + (r_1 - r_3)e^{(r_1 + r_3)sh} \\ + (r_3 - r_2)e^{(r_3 + r_2)sh} ,$$

$$m_{43}(s) = r_1(r_2 - r_3)e^{r_1 sh} - r_2(r_1 - r_3)e^{r_2 sh} + r_3(r_1 - r_2)e^{r_3 sh} ,$$

$$m_{44}(s) = (r_3 - r_2)e^{r_1 sh} + (r_1 - r_3)e^{r_2 sh} - (r_1 - r_2)e^{r_3 sh} . \quad (A19-A34)$$

APPENDIX B

Expressions of the kernels $k_{ij}(x_2, t)$, $(i, j=1, 2)$:

$$k_{ij}(x_2, t) = d_i \int_0^\infty [G_{ij}(x_2, t, s) + G_{ij}(x_2, t, -s)] ds, \quad (i, j) = (1, 2); \quad (B1)$$

$$d_1 = \frac{\omega_1 \omega_2}{2\pi(\omega_1 - \omega_2)}, \quad d_2 = \frac{1}{2\pi(\omega_1 - \omega_2)}; \quad (B2, B3)$$

$$G_{11}(x_2, t, s) = \frac{e^{-in_2 x_2 s}}{\Delta(s)} \left[h_1 E_1^1 + h_2 E_2^1 + h_3 E_1^2 + h_4 E_2^2 + h_5 \omega_1 F_1^1 \right. \\ \left. - h_5 \omega_2 F_2^1 + h_6 \omega_1 F_1^2 - h_6 \omega_2 F_2^2 \right],$$

$$G_{12}(x_2, t, s) = \frac{e^{-in_2 x_2 s}}{\Delta(s)} \left[-\omega_1 h_5 E_1^1 + \omega_2 h_5 E_2^1 - \omega_1 h_6 E_1^2 + \omega_2 h_6 E_2^2 \right. \\ \left. + \omega_1^2 h_1 F_1^1 + \omega_2^2 h_2 F_2^1 + \omega_1^2 h_3 F_1^2 + \omega_2^2 h_4 F_2^2 \right],$$

$$G_{21}(x_2, t, s) = \frac{e^{-in_2 x_2 s}}{\Delta(s)} \left[v_1 E_1^1 + v_2 E_2^1 + v_3 E_1^2 + v_4 E_2^2 + v_5 \omega_1 F_1^1 \right. \\ \left. - v_5 \omega_2 F_2^1 + v_6 \omega_1 F_1^2 - v_6 \omega_2 F_2^2 \right],$$

$$G_{22}(x_2, t, s) = \frac{e^{-in_2 x_2 s}}{\Delta(s)} \left[-\omega_1 v_5 E_1^1 + \omega_2 v_5 E_2^1 - \omega_1 v_6 E_1^2 + \omega_2 v_6 E_2^2 \right. \\ \left. + \omega_1^2 v_1 F_1^1 + \omega_2^2 v_2 F_2^1 + \omega_1^2 v_3 F_1^2 + \omega_2^2 v_4 F_2^2 \right]; \quad (B4-B7)$$

where the functions $E_j^k(x, t)$ and $F_j^k(s, t)$, $(j, k=1, 2)$ are given by equations (A10-A13), $\Delta(s)$ is given by (A18), and

$$h_1(x_2, s) = \sum_{k=1}^4 \alpha_k(x_2, s) [a_{1m_{k1}} + ia_{2m_{k2}}] ,$$

$$h_2(x_2, s) = \sum_{k=1}^4 \alpha_k(a_{3m_{k1}} + ia_{4m_{k2}}) ,$$

$$h_3(x_2, s) = \sum_{k=1}^4 \alpha_k(a_{1m_{k3}} + ia_{2m_{k4}}) ,$$

$$h_4(x_2, s) = \sum_{k=1}^4 \alpha_k(a_{3m_{k3}} + ia_{4m_{k4}}) ,$$

$$h_5(x_2, s) = \sum_{k=1}^4 \alpha_k [2n_1 n_2 m_{k1} - i(n_1^2 - n_2^2) m_{k2}] ,$$

$$h_6(x_2, s) = \sum_{k=1}^4 \alpha_k [2n_1 n_2 m_{k3} - i(n_1^2 - n_2^2) m_{k4}] ; \quad (B8-B13)$$

$$\alpha_k(x_2, s) = (n_1^2 r_k^2 - n_2^2 - 2in_1 n_2 r_k) e^{r_k n_1 x_2 s} , \quad (k=1, \dots, 4) ; \quad (B14)$$

$$v_1(x_2, s) = \sum_{j=1}^4 \beta_j(a_{1m_{j1}} + ia_{2m_{j2}}) ,$$

$$v_2(x_2, s) = \sum_{j=1}^4 \beta_j(a_{3m_{j1}} + ia_{4m_{j2}}) ,$$

$$v_3(x_2, s) = \sum_{j=1}^4 \beta_j(a_{1m_{j3}} + ia_{2m_{j4}}) ,$$

$$v_4(x_2, s) = \sum_{j=1}^4 \beta_j(a_{3m_{j3}} + ia_{4m_{j4}}) ,$$

$$v_5(x_2, s) = \sum_{j=1}^4 \beta_j [2n_1 n_2 m_{j1} - i(n_1^2 - n_2^2) m_{j2}] , \quad (B15-B20)$$

$$v_6(x_2, s) = \sum_{j=1}^4 \beta_j [2n_1 n_2 m_{j3} - i(n_1^2 - n_2^2) m_{j4}] ;$$

$$\beta_j(x_2, s) = [n_1 n_2 r_j^2 + n_1 n_2 + i(n_1^2 - n_2^2) r_j] e^{r_j n_1 x_2 s} , \quad (j=1, \dots, 4) \quad (B21)$$

$$a_1 = n_1^2 \omega_1 - \frac{n_2^2}{\omega_1} , \quad a_2 = \frac{n_1 n_2}{\omega_1} + n_1 n_2 \omega_1 ,$$

$$a_3 = -n_1^2 \omega_2 + \frac{n_2^2}{\omega_2} , \quad a_4 = -\frac{n_1 n_2}{\omega_2} - n_1 n_2 \omega_2 ; \quad (\text{B22-B25})$$

and the functions $m_{kj}(s)$, $(k,j=1,\dots,4)$ are given by equations (A19-A34).

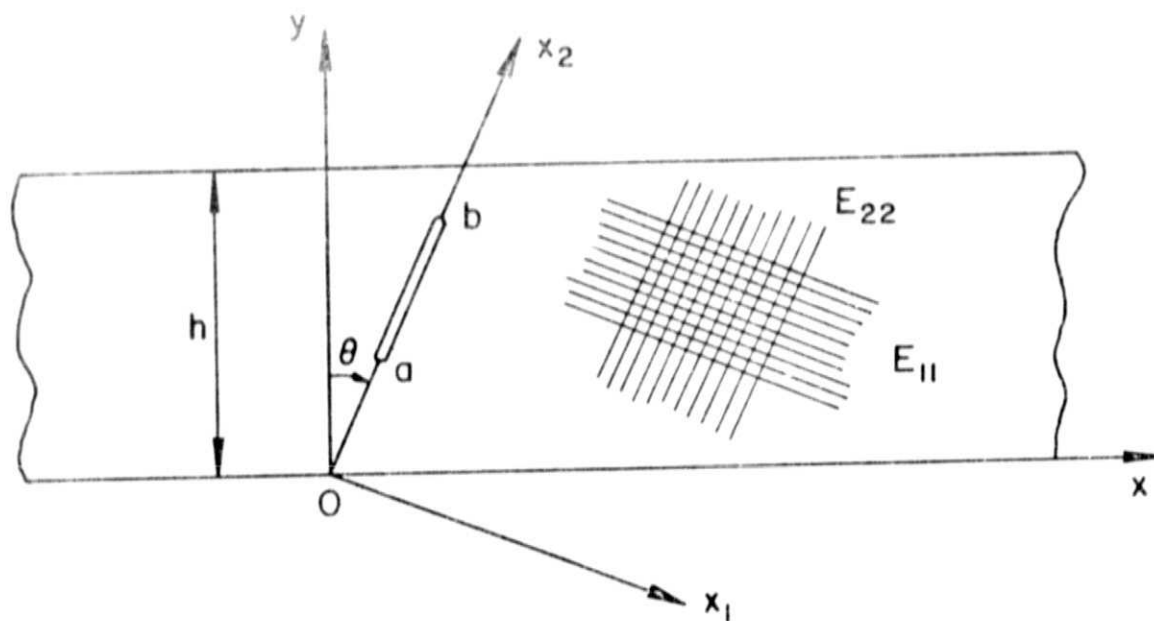


Figure 1. The geometry of orthotropic strip